



UNIVERSITY OF KRAGUJEVAC  
FACULTY OF MECHANICAL ENGINEERING KRALJEVO  
KRALJEVO - SERBIA

The Sixth Triennial  
International Conference

# HEAVY MACHINERY HM 2008

Proceedings

KRALJEVO  
24 - 29 June 2008



FACULTY OF MECHANICAL ENGINEERING KRALJEVO  
UNIVERSITY OF KRAGUJEVAC  
KRALJEVO – SERBIA

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THE SIXTH INTERNATIONAL TRIENNIAL CONFERENCE

# HEAVY MACHINERY HM 2008

PROCEEDINGS

ORGANIZATION SUPPORTED BY:

Ministry of Science, Republic of Serbia

KRALJEVO, 24. – 29. JUNE 2008.



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## PREFACE

The Faculty of Mechanical Engineering Kraljevo has been traditionally organizing the international scientific conference devoted to heavy machinery every three years. The VI International Scientific Conference HM 2008 is considering modern methods and new technologies in the fields of design in machinery, production technologies, urban engineering and QMS through thematic sessions for the purpose of sustainable competitiveness of economic systems. Modern technologies are exposed to fast changes at the global world level so that their timely application both in large industrial systems and in medium and small enterprises is of considerable importance for the entire development and technological progress of economy as a whole.

The VI International Scientific Conference Heavy Machinery HM 2008 is a place for exchange of experiences and results accomplished in domestic and foreign science and practice, with the goal to indicate directions of further development of our industry on its way toward integration in European economic trends. Exchange of experiences between our and foreign scientific workers should contribute to extension of international scientific-technical collaboration, initiation of new international scientific-research projects and broader international collaboration among universities.

The papers which will be presented at this Conference have been classified into three thematic fields. In the first thematic field: Machine Building Design, the scientific-research issues refer to:

- A. Automatic Control and Fluid Technique
- B. Earth-Moving and Transportation Mechanisation
- C. Railway Engineering
- D. Termotechnology, Environment Protection and Urban Engineering
- E. Mechanical Design and Mechanics
- F. Production Technologies, Material Application and Entrepreneurial Engineering and Management
- G. Computer-Integrated Processes and Designing of Machining Processes

Within this Conference, a round table with the topic "Energy Efficiency in Heavy Machinery" will be held. The aim is to open a scientific discussion on this actual problem in industry.

The sponsorship by the Ministry of Science of the Republic of Serbia is the proper way to promote science and technology in the area of mechanical engineering in Serbia.

On behalf of the organizer, I would like to express our thanks to all organizations and institutions that have supported this Conference. I would also like to extend our thanks to all authors and participants from abroad and from our country for their contribution to the Conference. And last but not the least, dear guests and participants in the Conference, I wish you a good time in Kraljevo – Mataruška Banja and see you again at the Seventh Conference, in three years.

Kraljevo, 19 June 2008

Conference Chairman,

Prof. Dr. Novak Nedić, mech eng.





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## IONIZED GAS BOUNDARY LAYER ON THE POROSITY WALL OF THE BODY WHOSE ELECTROCONDUCTIVITY IS A FUNCTION OF THE LONGITUDINAL VELOCITY GRADIENT

S. Savić, B. Obrović, M.Despotović

**Abstract:** Planar flow of the ionized gas in the boundary layer in the conditions of the so-called equilibrium ionization is studied. The contour of the body within the fluid is porous. The ionized gas is under the influence of the outer magnetic field which is normal to the contour of the body. The electroconductivity of the ionized gas is assumed to be a function of the longitudinal velocity gradient. In the paper, the governing equation system is with suitable transformations brought to a generalized form and numerically solved in the four-parametric approximation. Based on the obtained numerical solutions diagrams of important physical values and characteristics of the boundary layer are drawn. Adequate conclusions are also made.

**Key words:** boundary layer, ionized gas, ionized gas electroconductivity, porous contour, general similarity method, porosity parameter.

### 1. INTRODUCTION

This paper presents a detailed study of a complex ionized gas flow in the boundary layer along a porous contour. As known, at supersonic flow velocities the gas dissociation is followed by ionization. Hence, the gas becomes electroconductive. When the ionized gas is exposed to a magnetic field, an electric flow is formed in the gas. Due to this flow, the so-called Lorentz force and Joule heat generate. As a result, additional terms, which contain the gas electroconductivity, appear in the governing equations.

The most significant results in investigation of the dissociated gas flow are given in the book by Dorrance [1]. Loitsianskii and the members of his school [2, 3, 4, 5] performed a detailed investigation of the dissociated gas flow in the boundary layer. Investigators of the so-called Belgrade School of the Boundary Layer led by Saljnikov [6, 7, 8] accomplished significant results in the field of dissociated gas flow in the boundary layer. In the works of Boricic et al [9, 10, 11] and Ivanovic [12], MHD boundary layer on a porous and nonporous contour of a body within the fluid is studied. In the paper [2], the ionized gas flow in the boundary layer along a flat plate in the presence of a magnetic field is studied. The paper [13] studies the ionized gas flow in the boundary layer along a nonporous body and papers [14, 15] study the ionized gas flow along a porous body of an arbitrary shape. In these papers, different electroconductivity variation laws are used.

The presented paper gives the results of investigation of the ionized gas flow in the boundary layer along a porous wall in the case when the electroconductivity is a function of the longitudinal velocity gradient. The ionized gas of the same physical characteristics as in the

main flow is injected, i.e. ejected with the velocity  $v_w(x)$ . The outer magnetic field is normal to the wall of the body within the fluid. According to [2], it is considered that the power of this field is  $B_m = B_m(x)$  and that the magnetic Reynold's number is very small. Therefore, in the case of the ionized gas flow in the magnetic field, the governing equation system of steady planar laminar boundary layer with the corresponding boundary conditions, according to [2], takes the following form:

$$\begin{aligned} \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) &= 0, \\ \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} &= -\frac{dp}{dx} + \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) - \sigma B_m^2 u, \\ \rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} &= u \frac{dp}{dx} + \mu \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\partial}{\partial y} \left( \frac{\mu}{Pr} \frac{\partial h}{\partial y} \right) + \\ &+ \sigma B_m^2 u^2, \\ u = 0, \quad v = v_w(x), \quad h = h_w &\quad \text{for } y = 0, \\ u \rightarrow u_e(x), \quad h \rightarrow h_e(x) &\quad \text{for } y \rightarrow \infty. \end{aligned} \quad (1)$$

By analogy with MHD boundary layer [9], the ionized gas electroconductivity  $\sigma$  is assumed to be a function of the longitudinal velocity gradient

$$\sigma = \sigma_0 \frac{v_0}{u_e^2} \frac{\partial u}{\partial y}, \quad (\sigma_0, v_0 = \text{const.}) \quad (2)$$

If the pressure is eliminated from the system (1), based on the conditions for the outer edge of the boundary layer, the following equation system is obtained:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0,$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \rho_e u_e \frac{du_e}{dx} + \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) - \sigma B_m^2 u, \quad (3)$$

$$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = -u \rho_e u_e \frac{du_e}{dx} + \mu \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\partial}{\partial y} \left( \frac{\mu}{\text{Pr}} \frac{\partial h}{\partial y} \right) + \sigma B_m^2 u^2.$$

The boundary conditions remain unchanged.

## 2. TRANSFORMATION OF THE EQUATIONS

Modern methods of solution of boundary layer equations involve usage of a momentum equation. In the case of the ionized gas flow in the boundary layer along a porous wall, this equation will have the simplest form if instead of physical coordinates  $x$  and  $y$  we introduce new variables [3] in the form:

$$s(x) = \frac{1}{\rho_0 \mu_0} \int_0^x \rho_w \mu_w dx, \quad z(x, y) = \frac{1}{\rho_0} \int_0^y \rho dy \quad (4)$$

and a stream function  $\psi(s, z)$  by means of the relations:

$$u = \frac{\partial \psi}{\partial z}, \quad \tilde{v} = \frac{\rho_0 \mu_0}{\rho_w \mu_w} \left( u \frac{\partial z}{\partial x} + v \frac{\rho}{\rho_0} \right) = -\frac{\partial \psi}{\partial s}. \quad (5)$$

Quantities  $\rho_0$  and  $\mu_0$  denote the known values of the density and the dynamic viscosity of the ionized gas (air) at a concrete point.

Using transformations (4) and (5) the governing equation system, together with the boundary conditions, is transformed and brought to the following form:

$$\begin{aligned} \frac{\partial \psi}{\partial z} \frac{\partial^2 \psi}{\partial s \partial z} - \frac{\partial \psi}{\partial s} \frac{\partial^2 \psi}{\partial z^2} &= \frac{\rho_e}{\rho} u_e \frac{du_e}{ds} + v_0 \frac{\partial}{\partial z} \left( Q \frac{\partial^2 \psi}{\partial z^2} \right) - \\ &- \frac{\rho_0 \mu_0}{\rho_w \mu_w} \frac{\sigma_0 B_m^2 v_0}{\rho_0 u_e^2} \frac{\partial^2 \psi}{\partial z^2} \frac{\partial \psi}{\partial z}, \\ \frac{\partial \psi}{\partial z} \frac{\partial h}{\partial s} - \frac{\partial \psi}{\partial s} \frac{\partial h}{\partial z} &= -\frac{\rho_e}{\rho} u_e \frac{du_e}{ds} \frac{\partial \psi}{\partial z} + v_0 Q \left( \frac{\partial^2 \psi}{\partial z^2} \right)^2 + \\ + v_0 \frac{\partial}{\partial z} \left( \frac{Q}{\text{Pr}} \frac{\partial h}{\partial z} \right) &+ \frac{\rho_0 \mu_0}{\rho_w \mu_w} \frac{\sigma_0 B_m^2 v_0}{\rho_0 u_e^2} \frac{\partial^2 \psi}{\partial z^2} \left( \frac{\partial \psi}{\partial z} \right)^2; \quad (6) \end{aligned}$$

$$\frac{\partial \psi}{\partial z} = 0, \quad \frac{\partial \psi}{\partial s} = -\frac{\mu_0}{\mu_w} v_w = -\tilde{v}_w, \quad h = h_w \quad \text{for } z = 0,$$

$$\frac{\partial \psi}{\partial z} \rightarrow u_e(s), \quad h \rightarrow h_e(s) \quad \text{for } z \rightarrow \infty.$$

In the transformed equations (7), the nondimensional function  $Q$  and Prandtl number  $\text{Pr}$  are determined with the expressions:

$$Q = \frac{\rho \mu}{\rho_w \mu_w}, \quad \text{Pr} = \frac{\mu c_p}{\lambda}, \quad (7)$$

where  $\lambda$  is the thermal conductivity coefficient  $c_p$  - the specific heat of the ionized gas at a constant pressure.

In order to solve the equation system (4), it is necessary to derive the momentum equation of the ionized gas on a body with a porous contour

$$\frac{dZ^{**}}{ds} = \frac{F_{mp}}{u_e}. \quad (8)$$

While deriving the momentum equation, the usual quantities are introduced: a parameter of the form  $f(s)$ , magnetic parameter  $g(s)$ , a conditional displacement thickness  $\Delta^*$ , a conditional momentum loss thickness  $\Delta^{**}$ , a shear stress at the wall of the body within the fluid  $\tau_w$ , a nondimensional friction function  $\zeta(s)$ , a nondimensional value  $H$  and a characteristic boundary layer function on the porous wall  $F_{mp}$ . With the ionized gas flow, these quantities are defined with the relations:

$$\begin{aligned} Z^{**} &= \frac{\Delta^{**2}}{v_0}, \quad f(s) = f_1(s) = \frac{u_e' \Delta^{**2}}{v_0} = u_e' Z^{**}, \\ g(s) &= g_1(s) = u_e^{-1} N_\sigma v_0^{1/2} Z^{**1/2}, \quad N_\sigma = \frac{\rho_0 \mu_0}{\rho_w \mu_w} N, \\ N &= \frac{\sigma_0 B_m^2}{\rho_0}, \quad \Delta^*(s) = \int_0^\infty \left( \frac{\rho_e}{\rho} - \frac{u}{u_e} \right) dz, \\ \Delta^{**}(s) &= \int_0^\infty \frac{u}{u_e} \left( 1 - \frac{u}{u_e} \right) dz, \quad \tau_w(s) = \left( \mu \frac{\partial u}{\partial y} \right)_{y=0} = \\ &= \frac{\rho_w \mu_w}{\rho_0} \frac{u_e}{\Delta^{**}} \zeta; \quad \zeta(s) = \left[ \frac{\partial (u/u_e)}{\partial (z/\Delta^{**})} \right]_{z=0}, \end{aligned} \quad (9)$$

$$H = \frac{\Delta^*}{\Delta^{**}}; \quad F_{mp} = 2[\zeta - (2 + H)f] + g - 2\Lambda.$$

Due to the porous wall of the body within the fluid, an addend appears in the momentum equation. Therefore, it is necessary to introduce a new parameter, the so-called porosity parameter  $\Lambda(s)$ :

$$\Lambda = -\frac{\mu_0}{\mu_w} \frac{v_w \Delta^{**}}{v_0} = -\frac{V_w \Delta^{**}}{v_0} = \Lambda(s) \quad (10)$$

where

$$V_w(s) = \frac{\mu_0}{\mu_w} v_w = \tilde{v}_w.$$

In order to apply the general similarity method, it is very important that the boundary conditions and the stream function on the wall of the body within the fluid remain the same as with the nonporous wall. For that reason, a new stream function  $\psi^*(s, z)$  is introduced with the relation

$$\psi(s, z) = \psi_w(s) + \psi^*(s, z), \quad \psi^*(s, 0) = 0 \quad (11)$$

where  $\psi(s, 0) = \psi_w(s)$  denotes the stream function of the flow along the wall of the body within the fluid.

Applying the relation (11), the system (6) is transformed into the following equation system:

$$\begin{aligned} & \frac{\partial \psi^*}{\partial z} \frac{\partial^2 \psi^*}{\partial s \partial z} - \frac{\partial \psi^*}{\partial s} \frac{\partial^2 \psi^*}{\partial z^2} - \frac{d\psi_w}{ds} \frac{\partial^2 \psi^*}{\partial z^2} = \frac{\rho_e}{\rho} u_e u_e' + \\ & + v_0 \frac{\partial}{\partial z} \left( Q \frac{\partial^2 \psi^*}{\partial z^2} \right) - \frac{\sigma_0 B_m^2 v_0 \rho_0 \mu_0}{\rho_0 u_e^2 \rho_w \mu_w} \frac{\partial^2 \psi^*}{\partial z^2} \frac{\partial \psi^*}{\partial z}, \\ & \frac{\partial \psi^*}{\partial z} \frac{\partial h}{\partial s} - \frac{\partial \psi^*}{\partial s} \frac{\partial h}{\partial z} - \frac{d\psi_w}{ds} \frac{\partial h}{\partial z} = - \frac{\rho_e}{\rho} u_e u_e' \frac{\partial \psi^*}{\partial z} + \\ & + v_0 Q \left( \frac{\partial^2 \psi^*}{\partial z^2} \right)^2 + v_0 \frac{\partial}{\partial z} \left( \frac{Q}{Pr} \frac{\partial h}{\partial z} \right) + \\ & + \frac{\sigma_0 B_m^2 v_0 \rho_0 \mu_0}{\rho_0 u_e^2 \rho_w \mu_w} \frac{\partial^2 \psi^*}{\partial z^2} \left( \frac{\partial \psi^*}{\partial z} \right)^2; \\ & \psi^* = 0, \quad \frac{\partial \psi^*}{\partial z} = 0, \quad h = h_w \quad \text{for } z = 0, \\ & \frac{\partial \psi^*}{\partial z} \rightarrow u_e(s), \quad h \rightarrow h_e(s) \quad \text{for } z \rightarrow \infty. \end{aligned} \quad (12)$$

### 3. MATHEMATICAL MODEL

In order to derive the generalized boundary layer equations it is necessary to introduce new transformations from the very beginning:

$$\begin{aligned} s &= s, \quad \eta(s, z) = \frac{u_e^{b/2}}{K(s)} z, \\ \psi^*(s, z) &= u_e^{1-b/2} K(s) \Phi[\eta, \kappa, (f_k), (g_k), (\Lambda_k)], \\ h(s, z) &= h_1 \bar{h}[\eta, \kappa, (f_k), (g_k), (\Lambda_k)]; \\ h_e + \frac{u_e^2}{2} &= h_1 = const., \quad K(s) = \left( av_0 \int_0^s u_e^{b-1} ds \right)^{1/2}; \end{aligned} \quad (13)$$

$a, b = const.$

where  $\eta(s, z)$  is the newly introduced transversal variable,  $\Phi$  - the newly introduced stream function and  $\bar{h}$  - the nondimensional enthalpy. Some important quantities and characteristics of the boundary layer (10) can be written in the form of more suitable relations:

$$\begin{aligned} u &= u_e \frac{\partial \Phi}{\partial \eta}, \quad \Delta^{**}(s) = \frac{K(s)}{u_e^{b/2}} B(s), \\ B(s) &= \int_0^\infty \frac{\partial \Phi}{\partial \eta} \left( 1 - \frac{\partial \Phi}{\partial \eta} \right) d\eta, \end{aligned} \quad (14)$$

$$\frac{\Delta^{**}(s)}{\Delta^{**}(s)} = H = \frac{A(s)}{B(s)}, \quad A(s) = \int_0^\infty \left( \frac{\rho_e}{\rho} - \frac{\partial \Phi}{\partial \eta} \right) d\eta,$$

$$\zeta = B \left( \frac{\partial^2 \Phi}{\partial \eta^2} \right)_{\eta=0}, \quad \frac{f}{B^2} = \frac{a u_e'}{u_e^b} \int_0^s u_e^{b-1} ds.$$

In the general similarity transformations (13), with the nondimensional functions  $\Phi$  and  $\bar{h}$ , we introduced a local parameter of the ionized gas compressibility  $\kappa = f_0$ , a set of parameters of the form  $f_k$  of Loitsianskii's type [3], a set of magnetic parameters  $g_k$  and a set of porosity parameters  $\Lambda_k$  [16] by means of the following expressions:

$$\begin{aligned} \kappa &= f_0(s) = \frac{u_e^2}{2h_1}, \quad f_k(s) = u_e^{k-1} u_e^{(k)} Z^{**k}, \\ g_k(s) &= u_e^{k-2} N_\sigma^{(k-1)} v_0^{1/2} Z^{**k-1/2}, \\ \Lambda_k(s) &= -u_e^{k-1} \left( \frac{V_w}{\sqrt{v_0}} \right)^{(k-1)} Z^{**k-1/2} \quad (k=1, 2, 3, \dots). \end{aligned} \quad (15)$$

They present new independent variables that are used instead of the longitudinal variable  $s$ .

The local compressibility parameter  $\kappa = f_0$  and the sets of parameters satisfy the corresponding simple recurrent differential equations of the form:

$$\begin{aligned} \frac{u_e}{u_e'} f_1 \frac{d\kappa}{ds} &= 2 \kappa f_1 = \theta_0, \\ \frac{u_e}{u_e'} f_1 \frac{df_k}{ds} &= [(k-1)f_1 + kF_{mp}] f_k + f_{k+1} = \theta_k, \\ \frac{u_e}{u_e'} f_1 \frac{dg_k}{ds} &= (k-2)f_1 + \left(k - \frac{1}{2}\right) F_{mp} g_k + g_{k+1} = \gamma_k, \\ \frac{u_e}{u_e'} f_1 \frac{d\Lambda_k}{ds} &= \left\{ (k-1)f_1 + [(2k-1)/2] F_{mp} \right\} \Lambda_k + \\ &+ \Lambda_{k+1} = \chi_k. \quad (k=1, 2, 3, \dots) \end{aligned} \quad (16)$$

Applying the similarity transformations (13) and (15) to the equation system (12), we obtain the following boundary layer equation system:

$$\begin{aligned} & \frac{\partial}{\partial \eta} \left( Q \frac{\partial^2 \Phi}{\partial \eta^2} \right) + \frac{aB^2 + (2-b)f_1}{2B^2} \Phi \frac{\partial^2 \Phi}{\partial \eta^2} + \\ & + \frac{f_1}{B^2} \left[ \frac{\rho_e}{\rho} - \left( \frac{\partial \Phi}{\partial \eta} \right)^2 \right] - \frac{g_1}{B^2} \frac{\partial^2 \Phi}{\partial \eta^2} \frac{\partial \Phi}{\partial \eta} + \frac{\Lambda_1}{B} \frac{\partial^2 \Phi}{\partial \eta^2} = \\ & = \frac{1}{B^2} \left[ \sum_{k=0}^\infty \theta_k \left( \frac{\partial \Phi}{\partial \eta} \frac{\partial^2 \Phi}{\partial \eta \partial f_k} - \frac{\partial \Phi}{\partial f_k} \frac{\partial^2 \Phi}{\partial \eta^2} \right) + \right. \\ & \left. + \sum_{k=1}^\infty \gamma_k \left( \frac{\partial \Phi}{\partial \eta} \frac{\partial^2 \Phi}{\partial \eta \partial g_k} - \frac{\partial \Phi}{\partial g_k} \frac{\partial^2 \Phi}{\partial \eta^2} \right) + \right. \end{aligned} \quad (17)$$

$$\begin{aligned}
& + \sum_{k=1}^{\infty} \chi_k \left( \frac{\partial \Phi}{\partial \eta} \frac{\partial^2 \Phi}{\partial \eta \partial \Lambda_k} - \frac{\partial \Phi}{\partial \Lambda_k} \frac{\partial^2 \Phi}{\partial \eta^2} \right) \Bigg], \\
& \frac{\partial}{\partial \eta} \left( \frac{Q}{Pr} \frac{\partial \bar{h}}{\partial \eta} \right) + \frac{aB^2 + (2-b)f_1}{2B^2} \Phi \frac{\partial \bar{h}}{\partial \eta} - \frac{2\kappa f_1}{B^2} \frac{\rho_e}{\rho} \frac{\partial \Phi}{\partial \eta} + \\
& + 2\kappa Q \left( \frac{\partial^2 \Phi}{\partial \eta^2} \right)^2 + \frac{2\kappa g_1}{B} \frac{\partial^2 \Phi}{\partial \eta^2} \left( \frac{\partial \Phi}{\partial \eta} \right)^2 + \frac{\Lambda_1}{B} \frac{\partial \bar{h}}{\partial \eta} = \\
& = \frac{1}{B^2} \left[ \sum_{k=0}^{\infty} \theta_k \left( \frac{\partial \Phi}{\partial \eta} \frac{\partial \bar{h}}{\partial f_k} - \frac{\partial \Phi}{\partial f_k} \frac{\partial \bar{h}}{\partial \eta} \right) + \right. \\
& + \sum_{k=1}^{\infty} \gamma_k \left( \frac{\partial \Phi}{\partial \eta} \frac{\partial \bar{h}}{\partial g_k} - \frac{\partial \Phi}{\partial g_k} \frac{\partial \bar{h}}{\partial \eta} \right) + \\
& \left. + \sum_{k=1}^{\infty} \chi_k \left( \frac{\partial \Phi}{\partial \eta} \frac{\partial \bar{h}}{\partial \Lambda_k} - \frac{\partial \Phi}{\partial \Lambda_k} \frac{\partial \bar{h}}{\partial \eta} \right) \right].
\end{aligned}$$

The transformed boundary conditions are:

$$\begin{aligned}
\Phi = \frac{\partial \Phi}{\partial \eta} = 0, \quad \bar{h} = \bar{h}_w = \text{const.} \quad \text{for } \eta = 0, \\
\frac{\partial \Phi}{\partial \eta} \rightarrow 1, \quad \bar{h} \rightarrow \bar{h}_e = 1 - \kappa \quad \text{for } \eta \rightarrow \infty.
\end{aligned} \quad (18)$$

The generalized equation system (17) represents a general mathematical model of the ionized gas flow along a porous wall of the body within the fluid for the assumed form of the electroconductivity variation law.

#### 4. NUMERICAL SOLUTION

When the generalized equation system (17) with the boundary conditions (18) is numerically solved, a finite number of parameters should be adopted so that the solution is obtained in  $n$ -parametric approximation. The equation system can be solved only with a relatively small number of parameters. If we assume that all the similarity parameters from the second one onward equal zero:

$$\begin{aligned}
\kappa = f_0 \neq 0, \quad f_1 = f \neq 0, \quad g_1 = g \neq 0, \\
\Lambda_1 = \Lambda \neq 0; \quad f_2 = f_3 = \dots = 0, \quad g_2 = g_3 = \dots = 0, \\
\Lambda_2 = \Lambda_3 = \dots = 0,
\end{aligned} \quad (19)$$

the obtained equation system is significantly simplified. Furthermore, when the general similarity method is applied, the so-called localization is also performed. If we neglect derivatives per the compressibility, magnetic and porosity parameters, the equation system (17) is significantly simplified, and in a four-parametric three times localized approximation it has the following form:

$$\begin{aligned}
& \frac{\partial}{\partial \eta} \left( Q \frac{\partial^2 \Phi}{\partial \eta^2} \right) + \frac{aB^2 + (2-b)f}{2B^2} \Phi \frac{\partial^2 \Phi}{\partial \eta^2} + \\
& + \frac{f}{B^2} \left[ \frac{\rho_e}{\rho} - \left( \frac{\partial \Phi}{\partial \eta} \right)^2 \right] - \frac{g}{B^2} \frac{\partial^2 \Phi}{\partial \eta^2} \frac{\partial \Phi}{\partial \eta} + \frac{\Lambda}{B} \frac{\partial^2 \Phi}{\partial \eta^2} =
\end{aligned}$$

$$\begin{aligned}
& = \frac{F_{mp} f}{B^2} \left( \frac{\partial \Phi}{\partial \eta} \frac{\partial^2 \Phi}{\partial \eta \partial f} - \frac{\partial \Phi}{\partial f} \frac{\partial^2 \Phi}{\partial \eta^2} \right), \\
& \frac{\partial}{\partial \eta} \left( \frac{Q}{Pr} \frac{\partial \bar{h}}{\partial \eta} \right) + \frac{aB^2 + (2-b)f}{2B^2} \Phi \frac{\partial \bar{h}}{\partial \eta} - \\
& - \frac{2\kappa f}{B^2} \frac{\rho_e}{\rho} \frac{\partial \Phi}{\partial \eta} + 2\kappa Q \left( \frac{\partial^2 \Phi}{\partial \eta^2} \right)^2 + \frac{2\kappa g}{B} \frac{\partial^2 \Phi}{\partial \eta^2} \left( \frac{\partial \Phi}{\partial \eta} \right)^2 + \\
& + \frac{\Lambda}{B} \frac{\partial \bar{h}}{\partial \eta} = \frac{F_{mp} f}{B^2} \left( \frac{\partial \Phi}{\partial \eta} \frac{\partial \bar{h}}{\partial f} - \frac{\partial \Phi}{\partial f} \frac{\partial \bar{h}}{\partial \eta} \right).
\end{aligned} \quad (20)$$

The boundary conditions (18) remain unchanged.

In the equations of the system (20) the subscript 1 in some (first) parameters is left out.

For the numerical integration of the obtained system of differential partial equations of the third order, it is necessary to decrease the order of the differential equations. Using [7]

$$\frac{u}{u_e} = \frac{\partial \Phi}{\partial \eta} = \varphi = \varphi(\eta, \kappa, f, g, \Lambda), \quad (21)$$

we decrease the order of the differential equations of the system (20), so the system together with the boundary conditions comes to:

$$\begin{aligned}
& \frac{\partial}{\partial \eta} \left( Q \frac{\partial \varphi}{\partial \eta} \right) + \frac{aB^2 + (2-b)f}{2B^2} \Phi \frac{\partial \varphi}{\partial \eta} + \frac{f}{B^2} \left[ \frac{\rho_e}{\rho} - \varphi^2 \right] - \\
& - \frac{g}{B} \frac{\partial \varphi}{\partial \eta} \varphi + \frac{\Lambda}{B} \frac{\partial \varphi}{\partial \eta} = \frac{F_{mp} f}{B^2} \left( \varphi \frac{\partial \varphi}{\partial f} - \frac{\partial \Phi}{\partial f} \frac{\partial \varphi}{\partial \eta} \right), \\
& \frac{\partial}{\partial \eta} \left( \frac{Q}{Pr} \frac{\partial \bar{h}}{\partial \eta} \right) + \frac{aB^2 + (2-b)f}{2B^2} \Phi \frac{\partial \bar{h}}{\partial \eta} - \frac{2\kappa f}{B^2} \frac{\rho_e}{\rho} \varphi + \\
& + 2\kappa Q \left( \frac{\partial \varphi}{\partial \eta} \right)^2 + \frac{2\kappa g}{B} \frac{\partial \varphi}{\partial \eta} \varphi^2 + \\
& + \frac{\Lambda}{B} \frac{\partial \bar{h}}{\partial \eta} = \frac{F_{mp} f}{B^2} \left( \varphi \frac{\partial \bar{h}}{\partial f} - \frac{\partial \Phi}{\partial f} \frac{\partial \bar{h}}{\partial \eta} \right);
\end{aligned} \quad (22)$$

$$\begin{aligned}
\Phi = \varphi = 0, \quad \bar{h} = \bar{h}_w = \text{const.} \quad \text{for } \eta = 0, \\
\varphi \rightarrow 1, \quad \bar{h} \rightarrow \bar{h}_e = 1 - \kappa \quad \text{for } \eta \rightarrow \infty.
\end{aligned}$$

For the nondimensional function  $Q$  [15] and the density ratio  $\rho_e/\rho$  [4] that appear in the system (22), the following approximate formulae are used:

$$Q = Q(\bar{h}) = \left( \frac{\bar{h}_w}{\bar{h}} \right)^{1/3}, \quad \frac{\rho_e}{\rho} \approx \frac{\bar{h}}{1 - \kappa}. \quad (23)$$

A concrete numerical solution of the obtained system of nonlinear and conjugated differential partial equations (22) is performed using finite differences method, i.e., "passage method". Based on the scheme of the plane integration grid [7], derivatives of the functions  $\varphi$ ,  $\Phi$  and  $\bar{h}$  are substituted by finite differences ratios, and the equation system (22) is brought to the following system of algebraic equations:

$$\begin{aligned}
 (I) \quad & a_{M,K+1}^i \Phi_{M-1,K+1}^i - 2b_{M,K+1}^i \Phi_{M,K+1}^i + \\
 & + c_{M,K+1}^i \Phi_{M+1,K+1}^i = g_{M,K+1}^i, \\
 (II) \quad & a_{M,K+1}^j \bar{h}_{M-1,K+1}^j - 2b_{M,K+1}^j \bar{h}_{M,K+1}^j + \\
 & + c_{M,K+1}^j \bar{h}_{M+1,K+1}^j = g_{M,K+1}^j; \quad (24)
 \end{aligned}$$

$$\begin{aligned}
 M &= 2, 3, \dots, N-1; \quad K = 0, 1, 2, \dots; \quad i, j = 0, 1, 2, \dots \\
 \Phi_{1,K+1}^i &= \Phi_{1,K+1}^i = 0, \quad \bar{h}_{1,K+1}^j = \bar{h}_w = \text{const.} \quad \text{for } M = 1, \\
 \Phi_{N,K+1}^i &= 1, \quad \bar{h}_{N,K+1}^j = 1 - \kappa \quad \text{for } M = N.
 \end{aligned}$$

The equation system (24) consists of two subsystems - dynamic (I) and thermodynamic (II).

For the concrete numerical solution of the equation system (22), i.e., the corresponding algebraic system, a program in FORTRAN program language has been written. It is based on the program used in the investigations [7]. Since Prandtl number depends little on the temperature, for air, it is assumed to be:  $Pr = 0.712$ . The constants  $a$  and  $b$ , according to [7], have optimal values:  $a = 0.4408$ ;  $b = 5.7140$ .

As the equation system (22) is localized per the compressibility, porosity and magnetic parameters, these parameters have become simple parameters. Therefore, the equation system (22) is solved by the usual procedure starting from the value  $f = 0.00$  (flat plate), for values of the parameters  $\kappa$ ,  $g$  and  $\Lambda$  given in advance.

### 5. CONCLUSION

Only some of the results are presented in this paper in the form of diagrams based on which important conclusions are drawn:

- Regardless of the fact whether the ionized gas is injected into the main flow or ejected from it, at different cross-sections of the boundary layer, the nondimensional velocity  $u/u_e$  very quickly converges towards unity (Fig. 1).
- The magnetic field has a great influence upon the boundary layer characteristic  $F_{mp}$  (Fig. 2).
- The influence of the magnetic field on the nondimensional friction function  $\zeta$ , and therefore on the boundary layer separation point, is especially pointed out (Fig. 3). By increasing the values of the magnetic parameter, the separation of the boundary layer is postponed.
- The porosity parameter  $\Lambda$  has a great influence on the nondimensional friction function  $\zeta$  (Fig. 4). Consequently, this parameter has also a significant influence on the boundary layer separation point. It is noted that the injection of air, in accordance with the relation (10), postpones the separation of the ionized gas boundary layer because the separation point moves down the flow.

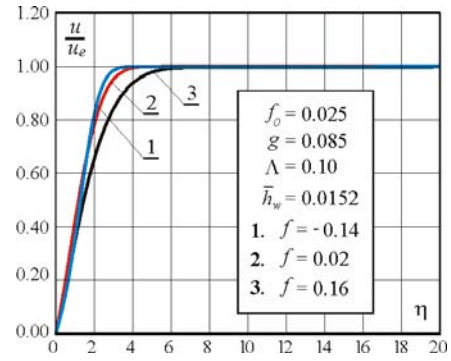


Fig. 1. Diagram of the nondimensional velocity  $u/u_e$

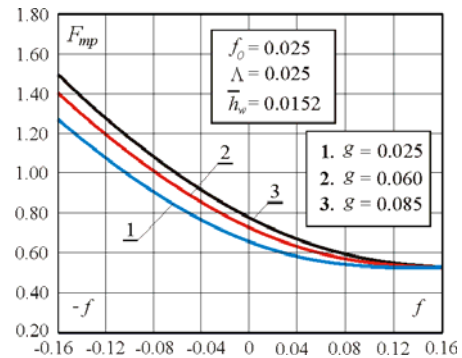


Fig. 2. Distribution of the characteristic function  $F_{mp}$

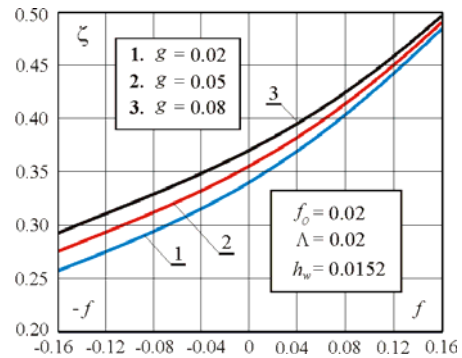


Fig. 3. Distribution of the nondimensional friction function  $\zeta(g)$

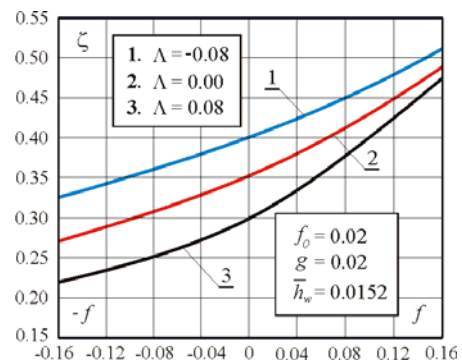


Fig. 4. Distribution of the nondimensional friction function  $\zeta(\Lambda)$

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